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Inferring solar differential rotation and viscosity via passive imaging with inertial waves

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Abstract

The recent discovery of inertial waves on the surface of the Sun offers new possibilities to learn about the solar interior. These waves are long-lived with a period on the order of the Sun rotation period (∼27 days) and are sensitive to parameters deep inside the Sun. They are excited by turbulent convection, leading to a passive imaging problem. In this work, we present the forward and inverse problem of reconstructing viscosity and differential rotation on the Sun from cross-covariance observations of these inertial waves.

Keywords: inverse problems, passive imaging, inertial waves, partial differential equations.

1 Introduction

Helioseismology aims at recovering parameters in the solar interior from surface observations using mostly acoustic *modes* (eigenfunctions). These modes have maximum sensitivity close to the surface and inferring deep inside the Sun is an extremely difficult task. The recent discovery of many inertial modes [2] opens up new opportunities for helioseismology. A linearized analysis of purely toroidal (i.e. divergence-free) modes on the surface of the Sun can already explain several observed features, such as the eigenfrequencies and eigenfunctions of Rossby modes and of high-latitude modes [1]. We use this forward model with an additional source to take into account the stochastic excitation of the waves, and solve an inverse problem to recover the solar differential rotation and the viscosity.

As this source is stochastic, we approach this from a *passive imaging* perspective; that is, we replace the source by its auto-correlation, which is an accessible entity known as the *source strength*, leading to a higher-dimensional problem and exacerbated nonlinearity, for which special strategies are required.

2 Solar inertial waves

We consider the dynamic of a moving particle in an incompressible fluid. Its velocity $\mathbf{v} = \mathbf{v}(\mathbf{r}, t)$ is described by the Navier-Stokes equation

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -\nabla p + \nabla\cdot(\rho\gamma\boldsymbol{\tau}) + \mathbf{f} - 2\boldsymbol{\Omega}_{\text{ref}}\times\mathbf{v}.$$

The restoring forces on the right hand side include acoustic pressure p, viscous stress $\boldsymbol{\tau} := \nabla\mathbf{v} + (\nabla\mathbf{v})^T$ and external source $\mathbf{f}$ (e.g. random source); ρ denotes density. The Coriolis force ($2\boldsymbol{\Omega}_{\text{ref}}\times\mathbf{v}$) results from observing the Sun in a rotating reference frame, often chosen as the Carrington frame with the angular velocity $\Omega_{\text{ref}} = 14.7^0/\text{days}$.

We decompose $\mathbf{v}$ into a background flow $\mathbf{u}_0$ and a perturbation (wave field) $\mathbf{u}$ in spherical coordinates

$$\mathbf{v} = \mathbf{u}_0 + \mathbf{u}, \quad \mathbf{u}_0 := (\Omega(r,\theta) - \Omega_{\text{ref}})[0; 0; r\sin\theta],$$

and suppose that the fluid is strongly stratified [3]. Considering the horizontal motion and expressing it via the stream function Ψ yields

$$\rho\mathbf{u} = \nabla\times\boldsymbol{\Psi}, \quad \boldsymbol{\Psi} := [\Psi(t,\theta,\phi); 0; 0].$$

In the frequency-latitude wave number domain, i.e. for each (ω, m), the inertial oscillation satisfies the scalar equation on $I := (0,\pi)$

$$\begin{aligned} &-\gamma\Delta_m^2\Psi - i\omega\Delta_m\Psi + im\beta\Delta_m\Psi - im\alpha\Psi = f \\ &\text{with } \gamma\in\mathbb{R}, \quad \beta(\theta) := \Omega(\theta) - \Omega_{\text{ref}} \quad \theta\in I \\ &\alpha(\theta) := \frac{1}{r^2\sin\theta}\frac{d}{d\theta}\left(\frac{1}{\sin\theta}\frac{d}{d\theta}(\Omega(\theta)\sin^2\theta)\right), \\ &\Delta_m\Psi := \frac{1}{r^2\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d}{d\theta}\Psi\right) - \frac{m^2}{r^2\sin^2\theta}\Psi, \end{aligned} \tag{1}$$

where Δ_m is the Laplace-Beltrami operator on the sphere. Supposing that the flow is continuous at the poles, we endow at the boundaries

$$\Psi(0) = \Psi(\pi) = 0, \quad \Psi'(0) = \Psi'(\pi) = 0. \tag{2}$$

The system (1)-(2) is the underlying model for the inverse problem of recovering viscosity γ and differential rotation β. Equation (1) reduces to that studied in [3] if $\gamma = 0$ and in [1] if $f = 0$.

3 Correlation-based passive imaging

On the Sun, the source of excitation f is passive, i.e. ambient noise near the surface of the convection zone. Consequently, the oscillation Ψ is a realization of a random wave. If f is spatially uncorrelated with $\mathbb{E}[f] = 0$, then its cross-correlation between two locations is

$$\mathbb{C}\mathrm{ov}[f, f](\theta, \theta') = \mathbb{E}[f(\theta)\overline{f(\theta')}] =: \Pi_f(\theta)\delta(\theta - \theta'),$$

where $\overline{(\cdot)}$ denotes the complex conjugate, δ is the Dirac function, and Π_f represents the source strength. In this spirit, averaging J correlated wave fields acquired separately yields

$$\mathbb{C}\mathrm{ov}[\Psi, \Psi] \approx \mathrm{Cov}(\Psi)(\theta, \theta') := \frac{1}{J}\sum_{j=1}^{J} \Psi_j(\theta)\overline{\Psi_j(\theta')},$$

referred to as the *empirical reprocessed data*. It is important to note that the source term f is typically not available, with only the source strength Π_f assumed known.

We formulate the passive imaging problem for solar viscosity and differential rotation as

$$\text{Find } (\gamma, \beta) \in \mathbb{R} \times X : \quad \mathrm{Cov}(\Psi) = y^{\mathrm{obs}}$$
$$\text{s.t. } \Psi \text{ solves (1)-(2)}$$

given Π_f and noisy observed data y^{obs}.

4 Inversion numerical results

We implemented an accelerated Nesterov Landweber algorithm to simultaneously reconstruct the scalar viscosity γ and latitude-dependent rotation β from a single frequency-latitude wave number.

Figure 1 shows the reconstruction results (top) and the corresponding covariance images (bottom). We initiate the algorithm with $\beta_{\mathrm{init}} = 0$, i.e. no prior knowledge, and γ_{init} very far from the ground truth. Despite this, the reconstruction approximates the ground truth extremely well. The algorithm required 200 iterations over 2 seconds on an i7-1255U CPU (4.70 GHz).

Outlook. Going forward, we will perform imaging with solar-like parameters and real data acquired from the Helioseismic and Magnetic Imager on board the Solar Dynamics Observatory.

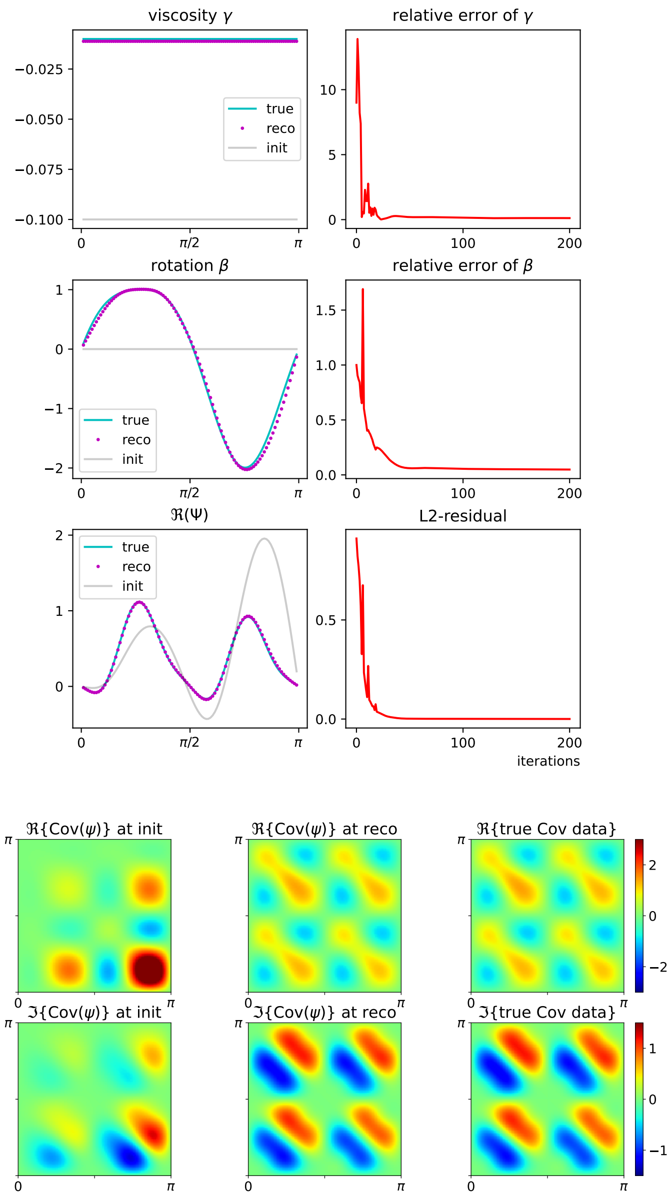

Figure 1: Simultaneous reconstruction of viscosity/rotation (top) and resulting covariance images (bottom).

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